Circuits: Symmetries

July 19, 2025

1 Background

In the last handout, we introduced some general concepts for solving circuit problems. Today, we will focus specifically on the utilization of symmetry in circuits.

What's so important about symmetry? Well, the more physics you do, the more important symmetry becomes - so let's start talking about it early! Identifiying symmetries lets us simplify problems, often by equating things that are in some way symmetric with each other. In the specific case of circuit problems. Identifying points that are symmetric to each other about some axis, lets us say that those points are at the same voltage. And this turns out to be very useful.

Using the tools that we outlined in the last handout, here is a framework for solving circuit problem with resistors:

1. Identify any symmetries in the circuit (Emil will expand on this in the session).
2. Based on these symmetries, deduce what points are at the same voltage. (And possibly identify points with no current between them)
3. Merge nodes with the same voltage, and open circuit nodes with no current between them (recall the node merging from our previous session).
4. This lets you simplify the circuit, often by relabeling the nodes and redrawing the circuit (also recall this from the previous session).

If much of this is unclear to you now, that's totally understandable! This explanation above is very high-level, and I don't expect you to have a clear idea of how it works just based on this. Let's start with some problems that hopefully clarify some of this.

2 Questions

2.1 Check your understanding

1. Let's do some initial reasoning about merging nodes to get in the spirit! Consider the circuit shown in figure 1.
   (a) What is the voltage difference between points A and B?
   (b) What is the current flowing from A to B?
   (c) Given your answer to b), what does this let you do with the circuit?
   (d) Confirm that the new circuit you obtained in c) gives you the same resistance as the one you started with.
2. (A repeat from the previous handout) Determine the resistance between the leads of the circuit in figure 2
3. Find the resistance between the centre point and one of the triangle's vertices in figure 3, utilizing the symmetry of the circuit.
4. Find the resistance of the circuit in figure 4. *Hint: Where is the current going?*

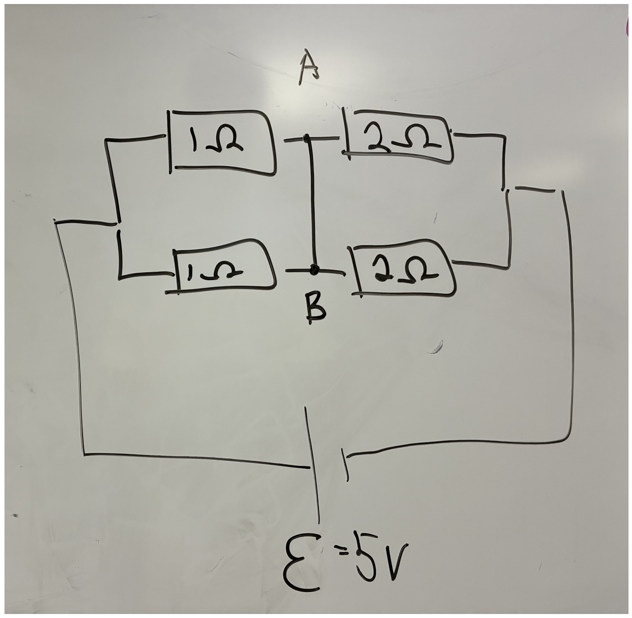

Figure 1: A nicely symmetrical circuit

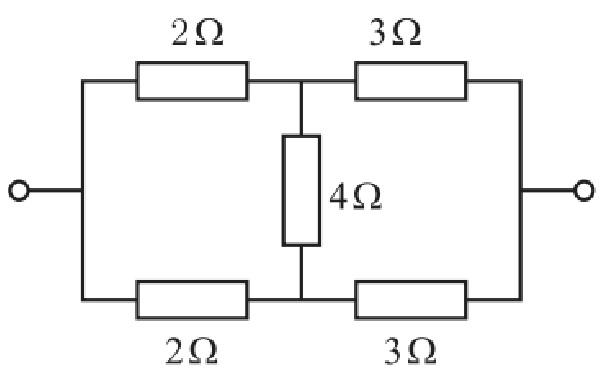

Figure 2: What points are symmetric about the axis defined by the voltage? What does this mean for their voltage?

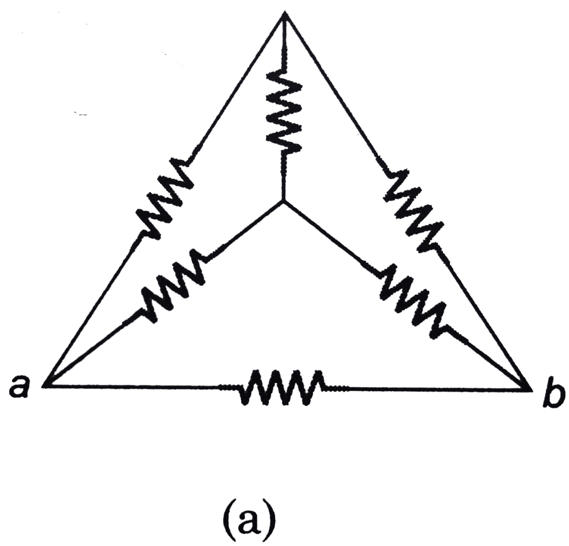

Figure 3: Triangle symmetric circuit

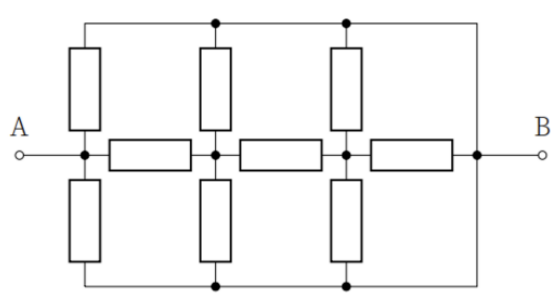

Figure 4: Where is all the current going to go?

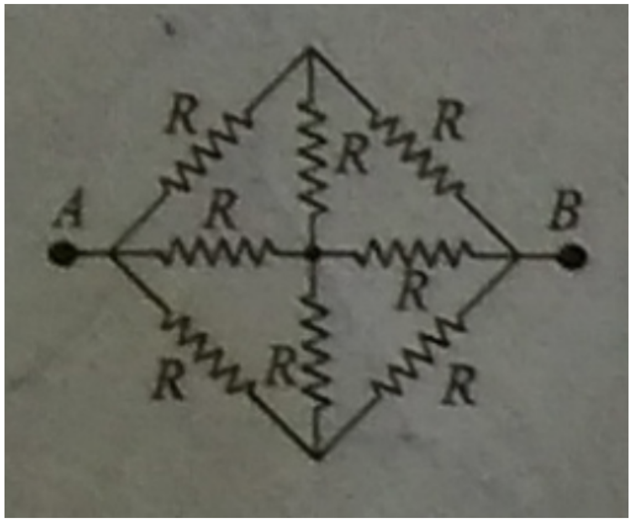

Figure 5: A small symmetric circuit

2.2 Trying out the waters

5. Find the resistance between points A and B in figure 5. *Hint: here, symmetry will help you, but won't get you all the way. If you remember the node labeling trick (trick 2) about redrawing circuits from last week's handout, this will help you here.*

6. Find the electrical resistance of a homogeneous wire frame in the shape of a regular hexagon with two diagonals linked together at O (figure 6). The current is led into the frame at the mid-points A and B of opposite sides of the hexagon. The resistance of one side of the hexagon is R. You may assume that the resistance of each half of every wire is R/2.

7. Find the resistance between points A and B in figure 7.

2.3 Exploring the deep

8. An hexagon ABCDEF with six "spokes" (connecting its centre O with the vertices) is made of 12 pieces of wire, each having a electrical resistance R. Find the resistance

   between the vertices A and O.

9. Figure **??** below depicts octahedron made from wire; the number near to each edge shows the resistance of the corresponding wire in ohms. The resistance of the wires connecting the ammeters are negligibly small. Find the readings of the ammeters.

10. There is an infinite honeycomb lattice (see figure 9); the edges of the lattice are made of wire, and the resistance of each edge is R. Let us denote two neighbouring vertices of a vertex B by A and C. Determine the resistance between:

    (a) A and B

    (b) A and C

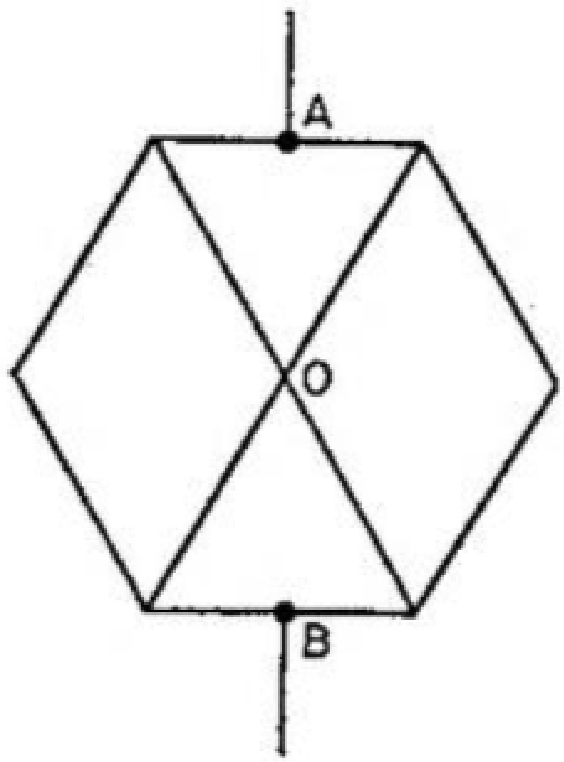

Figure 6: A rather symmetric hexagon

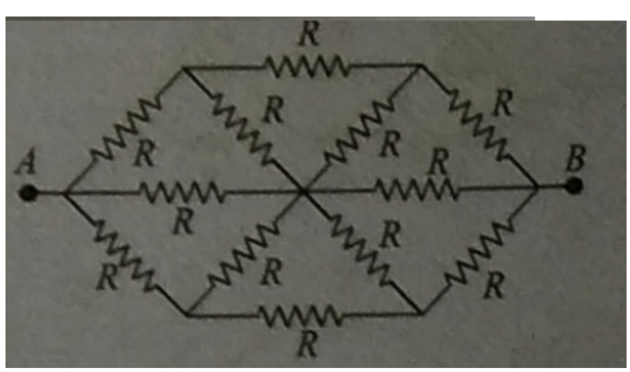

Figure 7: A larger symmetric circuit

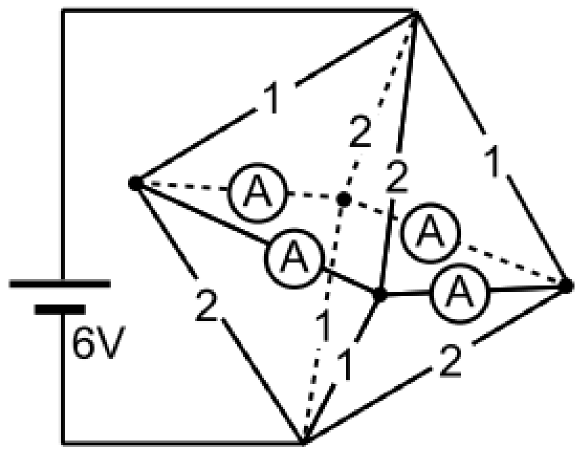

Figure 8: Current flowing through octahedron

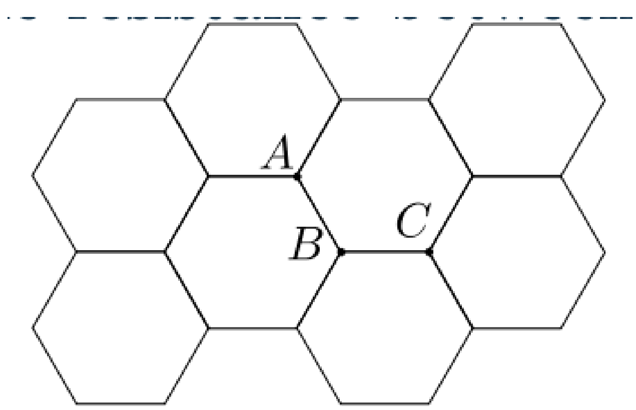

Figure 9: